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"How Many, How Much, and Who Cares?"
The Glorious Failure of Mathematics

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I. Mathematics and Reality

The relation between mathematics and philosophy today is an uneasy one, to say the least. As Penelope Maddy of UC Irvine says, "The philosophy of mathematics is . . . of fundamental importance to both mathematics and philosophy. Despite this, one finds surprisingly little co-operation between philosophers and mathematicians engaged in its pursuit; more often, widespread disregard and misunderstanding are broken only by pockets of outright antagonism." Both mathematics and philosophy have taken such a dramatic turn in the past few centuries – departing from the *status quo* they had enjoyed for millennia – that their vigorous repartee is inevitable. Philosophy claims responsibility for setting the other disciplines' houses in order, in virtue of its architectonic point of view; yet mathematics is now clearly the cultural paragon of knowledge, verity, and usefulness.

Richard Brown, mathematician at Johns Hopkins, defines mathematics as "the art of pure reason . . . the framework [providing] the rules of engagement for the entire system of structured thought . . . [helping] us to order and understand the very notion of everything we can imagine." And again: "We give the concepts in mathematics meaning only because they make sense and help us to order our existence. But outside of the meaning we give these elements of math, they do not really exist at all except in our imagination." If the first quote sounded hyperbolic, the second one is perhaps more sinister. Mathematics governs how we think about all of reality, and yet has *no more meaning* than what is arbitrarily imposed by our imaginations?

How did we arrive at this way of thinking, so counterintuitive? Well, Galileo and Descartes, more than anyone else, launched us on a new, mathematically-inspired way of looking at and understanding the world. The traditional philosophical analysis in terms of the four causes fell into desuetude as the vast potential of purely mathematical techniques, to say nothing of their scientific and technological implications, came into focus. Descartes especially would set the new tone and paradigm with his epochal shifting of mathematicians' emphasis from contemplation to operation, introducing methods that would make tractable mathematical "impossibilities" inherited from the centuries preceding. Construction, rather than discovery, would increasingly become the hallmark of mathematical labor – not merely in the classical sense of what can be done via "straightedge and compass," but that using any discoverable method. Mathematics would explode with such methods in the following centuries, with foundational contributions in analysis, calculus, number theory, topology, non-Euclidean geometries, set theory, and so on.

The late David Lachterman persuasively made the case, in *The Ethics of Geometry,* that "construction" has become the *Leitmotiv* of rationality itself: that "concepts, theories, systems, . . . even 'the world,' are all constructs, that is to say, fabrications . . . by the human intellect, the human will, or some tantalizing mélange of the two." He traces this state of affairs, which has become determinative of and for modernity, to "the deepest stratum of that 'Cartesian' soul" in which the seeds of Kant's philosophical idealism would be planted. A trajectory reaching from Kant to the twentieth-century positivists will display ever more insistence on intellect's *operation* – as opposed to its *receptivity* – in the act of knowing, especially mathematical knowing.

We are all too familiar now with a society in which there is scant advertence to objective reality: the subject has triumphed over the object, and with that triumph has come wholesale intellectual, moral, and cultural disorientation. Lachterman's contention is that this first began with, and has continued to model itself upon, the constructivist impulse in mathematics. Not that thinking in other disciplines has become mathematically informed (though that, too, is happening to a sometimes exorbitant degree, and is another symptom of modernity's malaise), but the power and the glory of mathematics lent early and ongoing credibility to the role of construction generally.

Today the dogma of constructionism is at least implied in many contexts. A well-known essay by the physicist Eugene Wigner echoes Einstein in raising a question about the "unreasonable effectiveness" of mathematics in the physical sciences. What is seen by Wigner as "unreasonable" – in the sense of "unexpectedly or mysteriously reasonable" – is not mathematics or physics as such, but the applicability of mathematics to the physical world. This "unreasonableness" implies the constructionist background I've been talking about. If we only construct mathematical entities and relations in our minds, how can they later turn out to describe physical phenomena that were previously unknown?

At least the "unreasonable effectiveness" idea implies also a world that is not *constituted* of quantities. Other recent thinkers have wandered further into realms of confusion, and assert that the sum total of physical reality is *none other than* number or quantity, in an unabashed, if unexcused, reversion to the ancient Pythagoras.

At Wyoming Catholic and similar liberal arts colleges, great value is placed upon the freshman introduction to mathematical reasoning via Euclid's *Elements*. And sentimental stories are sometimes heard about the world-weary person who, in a mid-life moment of crisis, happens to pull the dusty old forgotten volume of Euclid off the shelf, only to be wondrously transported back into a realm where truth is truth, conclusions follow inexorably from premises, and yes! there is a God above – a God who created all things in conformity with the fifth postulate!

What *is* mathematics, exerting such power over us? What, in particular, are the objects it studies – the "mathematicals," as philosophers sometimes call them? Another book used here at the College, Richard Courant's *What is Mathematics?* presents the elementary operations of mathematics with a lucidity that has kept it in print for seventy-five years now. But the book nowhere offers a direct

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answer to its titular question. It shows us how mathematics proceeds – which is indispensable to forming an idea of what it is – but it stops short of defining mathematics, or even of defining its proper object.

Courant assures us that "what points, lines, numbers 'actually' are cannot and need not be discussed in mathematical science." Of course these are philosophical questions, and Courant wrote as a mathematician. But it is disappointing that, in a book with so promising a title, he misses the mark entirely with his further comment that "it is not philosophy but active experience in mathematics itself that alone can answer the question."

So: are mathematical objects produced by the mind, or are they extramental things, *taken in* by the mind? Euclid, justly revered by mathematicians even to this day for his logical clarity, might seem a good place to start with our inquiry. Yet it is difficult to find evidence, in the *Elements,* for how Euclid conceives of mathematical objects, or what his philosophical views are at all. His formal logic is lauded through the ages, but his material logic is only implicit, left to us to figure out.

Lachterman argued, from Euclid's use of certain key verbs, that there is no basis for attributing to him a radical constructionism, that is, for supposing that in Euclid's mind the mathematicals exist only because he has been able to construct them. Rather – though the argument is admittedly subtle, as far as Euclid's text goes – the objects populating the *Elements* are simply exhibited through construction, so that their properties may be more easily seen from the principles thus exhibited.

As Lachterman says,

> The Kantian equation of constructability with the existence or objective reality of mathematical concepts . . . is not at home in the theoretical setting of the *Elements* . . . [T]he language for operations used by Euclid is almost always sensitive to the specific nature of the figure to be constructed, thereby reminding us that we must somehow be acquainted with that nature prior to any operations we perform.

Lachterman cites Archimedes's treatise *De sphaera et cylindro:* "These properties [of the mathematical objects] were by nature there in advance all along." "This," says Lachterman, "is precisely the claim that will be subverted and turned around by the radical moderns, for whom a figure has such and such properties and relations because it has been constructed in such and such a way."

II. Quantity

All roads in philosophy – in the realist tradition of *philosophia perennis* – proceed from Aristotle, and lead back to him. And "*sine Aquinate, silet Aristoteles*" – without Aquinas, Aristotle himself is mute. So what do these masters tell us about mathematics? Certainly this much, that mathematics is demonstrative knowledge of quantity through its properties. (This is a view that is being challenged by some today, but it remains the "naive realist" view, and the one that has held sway for over two millennia.) And what is quantity?

The mind's object, that in reference to which it functions, is *being*: more properly, it is being *qua* material. And material being, as the analyses of Aristotle and Aquinas make clear, exists in ten and only ten modes outside the mind; hence it has ten analogous ways of inhering in the intellect, so that there are ten conceptual, as well as real, categories. Of these, the first and fundamental one is substance*,* while the other nine – the so-called accidents – are modifications or aspects of substance, dependent on substance for their own existence (and conceptualization). The first of these secondary modes of being is *quantity*. It is prior to the other eight, in the sense of being existentially (and conceptually) presupposed by them. Quantity is proper to material beings and is a function of their materiality, rendering them *spatial*.

The categories cannot be properly defined, but only described – almost, as it were, by simply "pointing to them" – since there is no kind of being prior to them, in terms of which we might form definitions. Thus, quantity is first presented in Aristotle as that which answers our questions, "How many?" or "How much?"

Consider a line, and two different points on the line: what distinguishes these points is nothing other than their being "here" and "not-here." There is no other way to characterize the points; we don't say that one has more or less "pointness" than the other, or that it has any additional quality, such as shape, whereby we might distinguish it from the other. It is other simply in respect of its location in space – whether this is the real space of the cosmos, or the abstract space of the mathematical imagination.

The same can be said for entities in one, two, or three dimensions: their extendedness is what enables us to cognize parts differing from other parts solely in their spatial relationship. And if this pertains to the kind of quantity called continuous – the kind that answers our question, *How much?* in one, two, or three dimensions – it is no less true with respect to the kind that answers the question, *How many?* Discrete quantity may be described as not involving extension in its concept. But it is apprehended by us precisely *in virtue of* extension. Only spatial things, or things represented by us as spatial, can be counted. As Yves Simon observes in *The Great Dialogue of Nature and Space*,

> It is really, indeed, something intriguing and very puzzling. In quantity, you have a distinction of parts that are alike, homogeneous, of the same genus . . . and yet those parts are distinct, they are distinct by being outside each other. Here is the miracle of quantity. . . . [T]he word 'outside' is an appeal to a sensory and imaginative intuition, the intuition of outsidedness that suffices in itself.

If time allowed, I would trace the grounding of quantity in the substantial principle called prime matter – showing that extendedness is the very manifestation of materiality, so intimately bound up with materiality that brilliant Descartes could make the mistake of confusing them. Matter is seen in Aristotle's *Physics* to be the principle of motion and change; in his *Metaphysics* it is seen to be the principle of extension and individuation. These two accounts come together in the fact that it is extendedness that places bodies in adjacency, and it is as adjacent that they act upon one another, to effect changes directly accidental and indirectly substantial.

Just as quantity exists only in material substances with all their other accidents, so material substances don't exist without quantity. So much for how they exist, concretely. But in the order of thought there is no such entailment. We can conceptualize quantity in the abstract without doing violence to truth, because we are not claiming that quantity *exists* without sensible matter, only choosing to *consider* it without sensible matter.

III. Abstraction

All our knowledge is in a sense "unreal" – we know material things in an immaterial way, apprehending only their forms, as Aristotle says – though of course it is from reality itself that we do apprehend those forms, so that our knowledge is certainly a knowledge *of reality*. How does this occur? By what process does the human knower grasp *something real* about the object known, even while leaving aside that which characterizes the object as a real *thing*? It is called, not surprisingly, abstraction.

By way of approaching it, let's glance first at the somewhat different, though corresponding, process conceived by Platonists and Augustinians as *illumination*. Few things emerge more vividly from the Platonic tradition than the stark contrast between the world of material, changing things and the world of ideas, of things known with timeless verity. It was clear, to those in this tradition, that only an immutable, eternal principle could discover to us an immutable, eternal object – as the truths apprehended by intellect certainly appear to be. *Ergo*, there must be a direct illuminative activity within our souls, by none other than God Himself, whereby we are enabled to see, amid the flux of cosmic reality, truth and permanence.

Aquinas of course recognizes that every activity is ultimately caused by God, the unmoved Mover; but in following Aristotle's naturalist psychology he sees no need to posit a direct activity of God in the discernment of inner reality and truth: rather, for him the intellect includes, in its own God-given structure, an illuminating faculty whereby this occurs.

At stake here is something terribly important: something which, if you cannot adequately defend, becomes a mockery, a bitter absurdity at the heart of human experience and self-awareness, and you find that you have . . . well, the culture of 2016: a postmodern society no longer able to recognize human nature, having no access to enduring reality, no conviction of *meaning* anywhere at all. It is imperative that we have assurance that we are not trapped in our own mental constructs, that we develop and defend our insight into an external, mind-independent world. This entails that *something* in objective reality – something not ourselves – has to become internalized, a part of us.

The exterior senses take in material impressions; the brain forms images. But then, in humans, the image or phantasm is "made present" to a higher intellectual faculty (called the *agent intellect*) which acts on it in a manner proper to its own nature, that is to say, immaterially. Aquinas uses the term illumination for *this* process.

To illuminate is to enable to be seen what otherwise could not be seen, in accordance with the nature of the illuminating principle. Intellectual illumination implies discerning – by means of, but also quite beyond, the accidents of space and time – an underlying reality which is *not* radically constrained to "here" and "now," as are the material parts of the thing-as-sensed. The extendedness of its parts in space, and the distendedness of its parts through time, are magnitudes and multitudes grasped as a transcendent formal unity, by an intellectual principle that of its very nature transcends spatial and temporal constraints. Such unities cannot be grasped as such by lower animals, however keen their sensory powers. Only intellect transcends time itself, enabling the apprehension of future as future, and of final causality as the unifying cause of causes.

I am describing nothing less than what makes man *imago Dei*, “a little less than the angels” – this faculty of seeing time- and space-bound things in a perspective of formal discernment that is *not* time- and space-bound. Indeed, we know our own intellective soul to *be* immaterial, and therefore to be immortal, precisely *because* it can “become all things” in this supra-spatial, supra-temporal way. And let it be added, that when intellect is described as “separate” from the body we do not mean that it functions, in this life, *apart from* the body – notably, the brain and its phantasms – but rather that it is not a function *of* the brain.

IV. Mathematics

Intellectual abstraction, then, is a real, though immaterial, process, occurring in a real, immaterial entity (the soul), bringing about the existence, within that entity, of real, immaterial objects, modifications, acts of the soul – namely, *thoughts*. This is the essence of all human intellection. Now, there are further acts, whereby the mind can consider what it has primordially discerned in things, in yet more abstract ways. In fact, we can apprehend the quantitative aspect of a material object without considering anything sensible in that object.

Not only that, but we can consider quantity itself on more and more abstract levels, beginning with three-dimensionality (the abstraction closest to the way things really exist “out there”), proceeding to two-dimensionality (surfaces or planes), one-dimensionality (lines), and zero-dimensionality (giving us points, that is, “units having position”). Leaving out any consideration of dimensionality whatsoever, we have discrete quantity. But all of this abstraction is rooted in the intellect’s grasp of something in real beings. Aquinas, in his commentary on the *Metaphysics,* describes how mathematical objects are posited in the mind:

> [G]eometers discover the truth which they seek by dividing lines and surfaces. And division brings into actual existence the things which exist potentially. . . . Hence it is by making something actual that men attain knowledge, as is evident in [geometrical] constructions.

And again, with reference to discrete quantity:

> Number, formally speaking, is prior to continuous quantity, but materially speaking continuous quantity is prior, for number is the result of the division of the continuum.

Quantity, then, is real – in the sense that it is *taken from* the real. And yes, there is certainly a role for “construction,” not in the radical sense of modern subjectivism, but as a mental actuation consequent on abstraction from matter.

Let me anticipate an objection: what about . . . *Hobbits*? Their features are taken from reality – just because they are very short with big hairy feet doesn’t mean that Tolkien hasn’t drawn the notions of “short,” “big,” and “hairy” from the real world. Why do we say, Hobbits *cannot* be real, while mathematical abstractions are? It’s because Hobbits represent *our own assemblages* of features that we know very imperfectly, and which presumably could not be so assembled in reality. Mathematicals, on the other hand, are known to us in what I might call a "pure form," pure because abstracted, and posited, as complete wholes. We can in principle know their properties far more completely than we can know Hobbit-properties. Just think what would be implied by the notion of "Hobbit" as a potentially real being: we'd have to describe as completely knowable every last particular of its anatomy, physiology, biochemistry, genetics, developmental history, and so on!

Because it is an aspect of reality, quantity, even in the abstract, has a sort of nature, and properties flowing from that nature – as Archimedes said in the quotation I gave earlier. In fact, the science that infers quantitative properties from quantitative natures is precisely what mathematics *is*. It is the science of properties and relations, in so far as these are derived from the very essence of discrete and continuous quantity properly defined. These relations are complex and multi-layered. They include entities and operations that exist only to mediate other operations, or to secure consistency in other operations. But for the realist, such relations, however abstract, are not, and cannot be, entirely divorced from quantity as such.

The ongoing debate among philosophers, about whether mathematics pertains rather to quantity or to structure, would not have gotten too far in the pre-Cartesian world, when the mind's openness to external reality was still acknowledged. Quantity, I submit, comprises the material aspect of mathematical science; properties, structures, or relations comprise its formal aspect. However remote from the origins may seem the hyper-abstract mathematics of today, the fact remains that the unity and coherence of all mathematics – at least in so far as it has not become merely identified with *formal logic* – are ultimately tied to those quantitative foundations, even if they remain only implicit in the background.

The "effectiveness" of mathematics in the physical sciences may not be so "unreasonable" after all. Not that anyone is describing this "effectiveness" as perfect. Scientific journals witness every year to hundreds of shortcomings of math-in-science. Reasons aren't far to seek: leaving aside sources of human error, the sheer complexity of the world, grounded in the pluripotentiality of matter, must render our knowledge of it ever approximate. Aristotle noted that "nature acts [in the same way] always or for the most part," and he didn't mean that any formal principle of activity is fraught with inherent ambiguity – only that each nature is in conflict with other natures impinging unpredictably on its own sphere of activity.

The Heraclitean phrase "nature loves to hide" can be seen to mean (in Aristotelian terms) that we only infer formal principles, *i.e.,* natures, through ever-shifting configurations of accidents. Refined statistical methods enable scientists to minimize the resulting uncertainties; in fact, statistical "laws of nature" are increasingly replacing any attempt at more particularized descriptions of phenomena. But the underlying theory of probabilities remains an epistemological compromise.

One can certainly speak, then, of the limitations of mathematics in view of Nature's complexity. And it is prudence on the part of scientists to recognize those limitations, both within their respective disciplines and as regards science on the whole, in the face of cosmic ontological plenitude. But none of this betokens failure on the part of mathematics as such. (Nor, on the other hand, is there anything in what I've been describing that would merit the label "glorious" – but *stay with me!*)

Our "constructive" activity, to bring this back to those moderns I was complaining about, is not one of fashioning mathematical objects "from scratch," but is limited to our positing of entities in the mind, based on initial abstractions. And what is truly fascinating is that entities so posited prove to have such ramifications – that their far-reaching properties seem so disproportionate to what we derived from the outset. Surely this points to a God-given richness in quantitative reality itself.

I hope I've made a convincing case that mathematics is indeed about reality – only a part of reality, to be sure, but really a part; that it is "the truth, and nothing but the truth," even if it's not "the whole truth"; and that herein lies its applicability to scientific inquiry and to our world generally. But now I must try to unconvince you. It's only fair, given the complexity of our subject.

V. Truth

Let's begin with a fairly straightforward statement: "two plus two equals four." Is this statement true? I trust we can agree that this [I I] is what we designate as "two," and here [I I I I] is what we'll agree to call "four"; furthermore, we can all agree that "plus" means something like "taken together, as a whole." So, given these definitions, is it true that *two plus two make four*? Recall that truth is defined, in a realist philosophy, as a correspondence between what's inside the mind and what's outside, an *adequatio intellectus et rei.* So what is the *correspondence* here, if we're going to say that it is "true" that two and two make four?

Where do we find number? Is it "out there"? Here's a table with seven oranges on it: is "seven" located in the oranges? Certainly not in the sense that this orange is "number seven" – assigning an ordinal number to a given orange is clearly arbitrary, so that your seventh orange isn't necessarily my seventh orange. But is the *cardinal* number seven somehow attached to the oranges? They form a grouping, after all, which is really there in some sense regardless of whether I, or any other intellectual creature, is counting them. But what does it mean to say "they form a group"? Do the oranges "know" that they form a group – this group, rather than that one?

It is evident that number is the result of an *intellect arbitrarily considering*, that is, holding collectively in view, a certain group of objects. Number exists in the mind. And it is certainly therefore abstract. In counting oranges, I have just determined "this here" plurality. If there are also apples on the table, I know well enough not to count them, as long as "counting oranges" is the task at hand. If, on the other hand, I'm setting out to count *fruit*, then of course I include apples as well as oranges, and in fact I make no distinction between them. The point is, I make the determination, the distinction between oranges, apples, and fruit, in the very act of counting.

Now, no two oranges are exactly alike. *This* orange invariably differs from *that* one in its size and shape, in the number and arrangement of its component atoms – not to mention the fundamental fact that no two oranges, however else identical, can be in one and the same place, so they must be different in at least *that* respect. Each orange is existentially unique; and we cannot, strictly speaking, count what is unique.

Well, are we perhaps counting "oranges" as they exist in the mind? No: there is only one idea in the mind corresponding to "orange." It is an idea divested of precisely those individuating and multiplying aspects that would make "orange" a concrete existing individual, *instead of* an idea.

So it's neither "oranges out there" in their existential uniqueness, nor "orange as a single essence in the intellect," that we can be counting. Rather, when we count "seven oranges" we are in fact counting the idea of orange *as repeatedly instantiated* in objects "out there." Number, we say, exists only in the mind, but has its foundation in external reality. And only thus can we affirm that valid statements about number, like "2 + 2 = 4," are indeed true.

This criterion of truth – that what we hold in the mind must correspond to what is outside the mind – demands that we possess also some non-intellectual way of getting at "what's outside." Otherwise we're stuck in a vicious circle, validating what's in the mind only through what's in the mind, and we have thus embarked upon the post-Cartesian venture, which is very nearly a Satanic venture. The way out of such madness is to acknowledge the significance of our starting point, sense cognition, which is pre-rational. This does not mean, however, that we must refer all our thinking to sense objects *ad nauseam*, or that we can't build up elaborately abstract structures of thought, convinced of their truth-value in virtue of their inherent consistency and their conformity with the implicit foundations of all knowing.

To say, then, that "2 + 2 = 4" is *true* "purely in the abstract" is no more meaningful than to say that "Hobbits *truly* have big hairy feet." The latter statement is consistent within the abstract imaginative order that Tolkien conjured up, but consistency – or what logicians call validity – is not the same as truth, even if we occasionally broaden the use of the term "truth" to denote what is in fact *merely consistent* with certain other premises.

WCC FAQ

Why do our students study Math?

One of the more enduring schemas of the formative disciplines called "liberal arts" is that which divided them into the trivium, or verbal arts (grammar, logic, rhetoric), and the quadrivium, or mathematical arts (arithmetic, geometry, music, astronomy). Of these last four, the first two are in a sense "pure" and the other two "applied" – music employing arithmetic and astronomy employing geometry. Moreover, in the Platonic conception, music pertained to the inner cosmos, that constituting the human being, just as astronomy interpreted the outer cosmos, of which man is the mirror. Even today, we see that mathematics plays an indispensable role in understanding the physical universe of which we are a part.

What of geometry? It might seem a safe assumption that here again there is only a qualified sense in which our statements are "true." Or – is this case different? Geometrical objects, after all, appear to correspond more closely to their physical counterparts. We abstract solid, surface, line; we discern real properties and structural relations among them, establishing a science which turns out to have wonderful applicability to our world. And then a Lobachevski comes along and suddenly we're not so sure. Was our geometry true to begin with? In what sense was old Euclid "true" – that is, corresponding to what's outside the mind? Are there any straight lines in nature? Any perfect circles? We sure haven't *seen* them yet. Every straight line turns out to be crooked and bumpy at the atomic level. Every circular object we've experienced is to a degree *un*circular, with tiny little kinks and wobbles and even gaps.

This kind of thing attracted Plato's attention. He asked, how can we come up with the idea of "equality" (which really is the *main* idea in mathematics), if we have never encountered things truly equal? Well (you might say), we see things that are for all practical purposes equal. No! – mathematics is not about practical approximations. It is the paragon of exact science. Well (you might say again), we see things whose equality is apparent to us on a large scale, even if it disappears under magnification. But *that* seems to imply that mathematical equality was okay only as a sort of myth, until we grew up and learned to look more scientifically at our world.

Presumably, by analogy with what we said in connection with numbers, the ideas of "line, triangle" and so on do correspond to real aspects of things – and hence true statements can be made about them, inasmuch as these mental beings of geometry are somehow instantiated "out there." But since geometrical objects are, in a way, less abstract than numbers, they might present further difficulties. In numbering things, we're only focusing on the tiniest bit of intelligibility in them, and we gain certitude in so doing. (It's like saying we can have more certainty that there is *an animal* moving around out there in the fog, than that the animal is, in fact, *a dog* in the fog.)

Is geometry, then, less true, less certain in its applicability to the real world? I believe the answer to be: less applicable, yes, but not less true. In order to justify that opinion I must speculate – in a way that I will now present as the closing theme of this talk.

VI. Actuality or Reality?

Who should have thought, that any incompleteness in the applicability of mathematics to physical reality – for their correspondence is never quite perfect – should fall on the side of reality? Can it possibly be that our thoughts – derived from the encounter with Nature – are somehow too perfect to respresent that Nature?

Let us take a bold step and recognize, with Aristotle and all others in the hylomorphic tradition, that physical reality is form *and* matter, actuality *and* potentiality. Matter, though it be only a potency associated with form in the order of existents, is nonetheless a real principle of beings: never absent from them, rendering them ever liable to change, able to become other than they are. Regardless of how many present configurations of formal structure exist in the spatial continuum, indefinitely more of them remain possible. Unlike the case with Tolkien's Hobbits, or any product of our own fantasy, which could not be conceived so completely as to ensure its existibility in the physical realm, mathematical beings, in virtue of the mode of their abstraction, are already complete in their own order.

Nothing in the notion of a straight line, or of matter, precludes the line's being realized in matter, *even if* one has never yet been so realized. Nothing, in the definition and essence of a triangle, or a parallelepipedal solid, militates against its possible existence as the real boundary of a two- or three-dimensional physical object respectively; quite the contrary. (It is highly doubtful that we would even be able to recognize them as such if they did exist; there would always be room for doubt concerning our sense powers and so on.) But there is nothing – to my knowledge – in the order of mathematics, or of nature itself, that forbids the existence of such perfect entities.

It must be that somehow the intellect has abstracted, in the order of quantity, not only what *is*, in the field of natural objects, but what *can be.* We have seen, geometrically speaking, possibilities in the universe that have not even been realized by the Creator, and yet which *could be* realized by Him. Our perspective, if this is true, is divine. It is a perspective conformable to our spiritual nature, since only what transcends the material can view the material as material, as having potentialities beyond the here and now.

What is the connection between this "divine" discernment in regard to mathematicals and the discernment that I earlier attributed to all human knowing? I think they're essentially the same; the mind exhibits its transcendence of space and time in either case. The mind abstracts "perfect" mathematical natures from an imperfectly mathematical world because it is able to discern transcendentally, as a formal unity in the order of quantity, what the senses witness to only as disunified.

Realizable mathematical possibilities can be anticipated in the human mind, in a way that complete natures cannot, because of their restricted ontological status. Quantities are not creatures in their own right, but aspects of hylomorphic substance as such. We can "know" them prior to actual existence because this knowledge implies only the most generic conceptions of material being. (On the other hand, we cannot have a true science of other accidental kinds of being because the others inhere on the side of form, rather than matter, and are thus more dependent on natures fully constituted than on what simply conditions all such natures.)

I may seem to have come full circle: scorning modern philosophers' claim to construct all knowledge, I argued, with Aristotle and Aquinas, that knowledge is derived from extramental reality; yet now I've admitted that mathematical knowledge is in some crucial respects *not* referable to the actual world. I hereby admit, on behalf of mathematical humanity, to radical failure. Geometry, at least, has quite shockingly failed to "map onto" the external world.

This failure is, however, a *glorious* failure. It's not just, as we said earlier, that mathematical physicists have yet to "catch up" with the recalcitrant intricacies of the natural world. Mathematics itself, in some fundamental principled way, has proved *not* to correspond to the actuality of that world. Mathematics is "too perfect"; conversely, physical actuality is mathematically imperfect. "God's ways are not our ways," the godly man may be thinking. "Perhaps mathematics is just an excrescence of human fantasy." *No* . . . something so beautiful must be susceptible of truth, must be objectively founded, must be of God!

All we need to remember is that created reality is not limited to what is *actual*, just as the Creator's active potency is not limited to what He has already created. The passive potency called prime matter is as much a principle of cosmic being as is form or actuality. Somehow, mathematical knowledge leaps beyond what is given in the present actuality. Hyperbolas and logarithmic spirals and dodecahedra *are* "out there," if not actually, yet nonetheless really – that is, as *potentialities in real matter*. And it is as such that we can know them, abstract them, posit them in our imaginations, know their properties and structural relations, know them even before they exist – know them as "existibles" – in a way that truly speaks of divinity, of the imaging of God, in ourselves.

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